An extension of fuzzy TOPSIS approach using integral values for banking performance evaluation

Vu Thi Nhu Quynh

Abstract Banks play a crucial role as financial intermediaries in advancing economies. Increasing market share is a top priority for all banks in today’s fiercely competitive market. To achieve this, banks are expected to enhance their efficiency, which boosts their competitive capacity and helps decision-makers identify areas for development. This paper introduces an extended fuzzy TOPSIS approach that incorporates integral values to assess banking performance. Our method includes the averaging and normalization of decision-makers assigned ratings and weights, ensuring a standardized scale for comparison. Subsequently, we create membership functions for the ultimate fuzzy evaluation of alternatives by computing their distances to both the positive and negative ideal solutions. To illustrate the effectiveness and applicability of our proposed method, we introduce a closeness coefficient for determining the rank of alternatives. Our method introduces an extended fuzzy TOPSIS approach that incorporates integral values to assess banking performance. Our approach includes the averaging and normalization of decision-makers assigned ratings and weights, ensuring a standardized scale for comparison. Subsequently, we create membership functions for the ultimate fuzzy evaluation of alternatives by computing their distances to both the positive and negative ideal solutions. To illustrate the effectiveness and applicability of our proposed method, we present a novel fuzzy number ranking method based on integral values to streamline the aggregation of fuzzy numbers to defuzzify the weighted ratings. Ultimately, we introduce a closeness coefficient for determining the ranking order of alternatives by computing their distances to both the positive and negative ideal solutions. To illustrate the effectiveness and applicability of our proposed method, we present an application example.

Keywords: ranking fuzzy numbers, fuzzy TOPSIS, integral value, banking performance evaluation

1. Introduction

Banks are crucial organizations that contribute significantly to the economic development of a country. The banking sector’s active role affects other industries through banking activities, making it essential to assess their performance continuously (Akkoc and Vatansever, 2013). Banks operate in a constantly changing and fiercely competitive environment, and thus, it is essential for them to systematically assess their performance in order to bolster their competitiveness. Assessing the performance of banks is a multifaceted and vital task, necessitating adaptable and analytical approaches to address the multifaceted nature of this challenge (Tüysüz and Yıldız, 2020). Past data highlights that banks consistently seek to gauge their performance and gauge it against benchmark standards to assess the efficacy of their enhancement efforts while dealing with a variety of constraints (Guru and Mahalik, 2019).

It has been widely acknowledged in literature that evaluating banking performance is a multifaceted task that requires consideration of both qualitative and quantitative criteria. The criteria for evaluating banking performance encompass many factors, which include but are not limited to asset quality, earnings and profitability, and interest rate sensitivity (Akkoc and Vatansever, 2013; Shen and Tzeng, 2015). Considering the inherent intricacy and unpredictability of the decision-making environment, one can view the evaluation of banking performance as a multi-criteria decision-making (MCDM) challenge. Consequently, a multitude of studies have been conducted to address this issue. Wu et al. (2009) introduced a fuzzy MCDM approach and identified 23 out of 55 evaluation indices based on the perspectives of the balanced scorecard. Shaverdi et al. (2011) devised an approach rooted in MCDM and the balanced scorecard (BSC) to evaluate the performance of three non-governmental Iranian banks. They employed the fuzzy AHP to compute the relative weights of selected indices to account for the inherent vagueness and ambiguity in the available information. Akkoc and Vatansever (2013) conducted an assessment of the financial performance of twelve Turkish commercial banks by employing fuzzy AHP-TOPSIS method. Shen and Tzeng (2015) proposed an integrated soft computing model (DANP with VIKOR) to address the issue of predicting the financial performance of banks.

In the research conducted by Guru and Mahalik (2019), an integrated MCDM approach was employed to assess the efficiency of different public sector banks in India. AHP was utilized to establish the weight criteria. Tüysüz and Yıldız (2020) introduced a novel hybrid multi-criteria performance evaluation model tailored for the banking sector. Wang et al. (2020) tackled the evaluation of innovation performance within the Turkish banking industry by identifying eight distinct financial and non-financial criteria. Their method encompassed the application of Interval Type 2 Fuzzy DEMATEL to weigh the various dimensions. The fuzzy VIKOR approach was then used to rank the alternatives. Yazdı et al. (2020) conducted an evaluation of...
the performance of Colombian banks employing hybrid MCDM methods. Eighteen performance indicators were extracted, with six performance indicators selected from each of the four perspectives, namely financial, customer, internal processes, and growth and learning. The step-wise weight assessment ratio analysis was employed to rank these performance indicators. In a study by Daiy et al. (2021), a comparative analysis of the performance of selected private and public sector banks in India over a five-year period was undertaken. While there is a diversity of methods available for assessing banking performance, it is important to note that many of the approaches mentioned above do not offer the capability to derive defuzzification formulas from the membership functions of the final evaluation values. This limitation can constrain the applicability of existing fuzzy MCDM methods.

In recent years, the TOPSIS (Hwang and Yoon, 1981) has gained prominence as a favored approach for addressing MCDM challenges. Recent applications of TOPSIS can be found in several notable studies (Yang et al., 2022; Wang et al., 2022; Makwakwa et al., 2023; Hajiajhaei-Keshteli et al., 2023; Jin, 2023; Fu et al., 2023; de Lima Silva et al., 2023). In the context of this paper, we introduce a novel Fuzzy TOPSIS methodology designed specifically for evaluating banking performance. This innovative method allows for the representation of the assessments for each alternative and the respective importance assigned to each criterion by decision-makers in the form of fuzzy numbers. To simplify the complex process of aggregating these fuzzy numbers, we then transform these weighted ratings into precise values using a newly introduced ranking technique. The ranking order of the alternatives is established by a closeness coefficient, which computes the distances of these alternatives from both the ideal and negative-ideal solutions. To illustrate the applicability and efficiency of the proposed method, a banking performance evaluation problem is presented and addressed.

In Vietnam, the banking sector is categorized into three primary groups: state-owned banks, private banks, and foreign banks. State-owned banks primarily offer financial services to government entities, organizations, and state-owned enterprises. Private banks, on the other hand, contribute significantly to providing financial support for businesses and individuals nationwide. Foreign banks typically have large investment capital, advanced technology, and high-risk management experience. From 2018 to 2023, banks in Vietnam have continuously improved the quality of services, strengthened risk management, and provided solutions to support businesses and start-ups. Vietnamese banks are currently facing many challenges, including fierce competition from new banks, technological advancements, and increasing demand for diverse financial services. Based on the prevailing evaluation criteria in the literature, this manuscript employs five criteria to evaluate banking performance, encompassing capital sufficiency, asset quality, earnings and profitability, liquidity, and interest rate sensitivity (Akkoc and Vatansever, 2013; Shen and Zheng, 2015).

The subsequent sections of this paper are structured as follows. Section 2 provides an in-depth review of the fundamental concepts pertaining to fuzzy numbers. In Section 3, a novel fuzzy TOPSIS approach is introduced, employing the integral ranking method. Section 4 demonstrates the practical application of the proposed fuzzy TOPSIS approach in resolving the banking performance evaluation problem. The paper is rounded off with conclusive remarks presented in Section 5.

2. Preliminaries

The fundamental concepts of fuzzy numbers and their arithmetic operations are defined as follows:

**Definition 1.** The membership functions \( f_T^l(x) \) of the trapezoidal fuzzy number \( T \) can be expressed as:

\[
f_T^l(x) = \begin{cases} 
  f_T^l(x), & \tilde{a}_1 \leq x \leq \tilde{a}_2, \\
  1, & \tilde{a}_2 \leq x \leq \tilde{a}_3, \\
  f_T^r(x), & \tilde{a}_3 \leq x \leq \tilde{a}_4, \\
  0, & \text{otherwise}, 
\end{cases}
\]

where \( f_T^l(x) \) and \( f_T^r(x) \) are the left and right membership functions of \( T \), respectively. When \( \tilde{a}_2 = \tilde{a}_3 \), the trapezoidal fuzzy number is reduced to a triangular fuzzy number and can be denoted by \( A = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_4) \).

**Definition 2.** \( \alpha \)-cuts

The \( \alpha \)-cuts of a fuzzy number \( T \) can be defined as \( T^\alpha = \{ x \mid f_T(x) \geq \alpha \} \), \( \alpha \in [0, 1] \), where \( T^\alpha \) is a non-empty bounded closed interval in \( R \) and can be denoted by \( T^\alpha = [T^{\alpha L}, T^{\alpha U}] \), \( T^{\alpha L} \) and \( T^{\alpha U} \) are its lower and upper bounds, respectively.

**Definition 3.** Arithmetic operations on fuzzy numbers
Given two fuzzy numbers $\tilde{T}_1$ and $\tilde{T}_2$, where $\tilde{T}_1, \tilde{T}_2 \in \mathbb{R}^*$, the $\alpha$-cuts of $\tilde{T}_1$ and $\tilde{T}_2$ are $\tilde{T}_1^\alpha = [\tilde{T}_1^{\alpha}, \tilde{T}_1^{\alpha\alpha}]$ and $\tilde{T}_2^\alpha = [\tilde{T}_2^{\alpha}, \tilde{T}_2^{\alpha\alpha}]$, respectively. By the interval arithmetic, some main operations of $\tilde{T}_1$ and $\tilde{T}_2$ are defined as follows:

\[
\left(\tilde{T}_1 \oplus \tilde{T}_2\right)^\alpha = [\tilde{T}_1^{\alpha} + \tilde{T}_2^{\alpha}, \tilde{T}_1^{\alpha\alpha} + \tilde{T}_2^{\alpha\alpha}]
\]

(2)

\[
\left(\tilde{T}_1 - \tilde{T}_2\right)^\alpha = [\tilde{T}_1^{\alpha} - \tilde{T}_2^{\alpha}, \tilde{T}_1^{\alpha\alpha} - \tilde{T}_2^{\alpha\alpha}]
\]

(3)

\[
\left(\tilde{T}_1 \otimes \tilde{T}_2\right)^\alpha = [\tilde{T}_1^{\alpha} + \tilde{T}_2^{\alpha}, \tilde{T}_1^{\alpha\alpha} + \tilde{T}_2^{\alpha\alpha}]
\]

(4)

\[
\left(\tilde{T}_1 / \tilde{T}_2\right)^\alpha = [\tilde{T}_1^{\alpha} / \tilde{T}_2^{\alpha}, \tilde{T}_1^{\alpha\alpha} / \tilde{T}_2^{\alpha\alpha}]
\]

(5)

\[
\left(\tilde{T} \otimes r\right)^\alpha = [\tilde{T}^{\alpha} r, \tilde{T}^{\alpha\alpha} r], r \in \mathbb{R}^*
\]

(6)

3. Model establishment

To address the banking performance evaluation challenge, we have devised a novel fuzzy TOPSIS model. In this context, a committee comprising $l$ decision makers $(D_t, t = 1, \ldots, l)$ tasked with assessing $m$ alternatives $(A_i, i = 1, \ldots, m)$ under a set of $n$ evaluation criteria $(C_j, j = 1, \ldots, n)$, wherein both the suitability ratings for each alternative under every criterion and the criteria weights are expressed in linguistic terms. The process of our proposed TOPSIS method can be broken down into the following steps:

3.1. Aggregate ratings of alternative versus criteria

Let $\tilde{x}_{ij} = (\tilde{e}_{ij}, \tilde{f}_{ij}, \tilde{g}_{ij})$, $i = 1, \ldots, m; j = 1, \ldots, n; t = 1, \ldots, l$, be the suitability rating assigned to alternative $\tilde{A}_i$ by decision maker $D_t$, for criterion $\tilde{C}_j$. The averaged suitability rating, $\tilde{x}_{ij} = (\tilde{e}_{ij}, \tilde{f}_{ij}, \tilde{g}_{ij})$, can be evaluated as:

$$\tilde{x}_{ij} = \frac{1}{l} \left( \tilde{e}_{ij1} \oplus \tilde{e}_{ij2} \oplus \ldots \oplus \tilde{e}_{ijl} \oplus \ldots \oplus \tilde{e}_{ijl} \right),$$

(7)

where $\tilde{e}_{ij} = \frac{1}{l} \sum_{t=1}^{l} \tilde{e}_{ijt}$, $\tilde{f}_{ij} = \frac{1}{l} \sum_{t=1}^{l} \tilde{f}_{ijt}$, and $\tilde{g}_{ij} = \frac{1}{l} \sum_{t=1}^{l} \tilde{g}_{ijt}$.

3.2. Aggregate the importance weights

Let $\tilde{w}_j = (\tilde{o}_j, \tilde{p}_j, \tilde{q}_j)$, $\tilde{w}_j \in \mathbb{R}^*$; $j = 1, \ldots, n; t = 1, \ldots, l$ be the weight assigned by decision maker $D_t$ to criterion $\tilde{C}_j$. The average weight, $\tilde{w}_j = (\tilde{o}_j, \tilde{p}_j, \tilde{q}_j)$, of criterion $\tilde{C}_j$ assessed by the committee of $l$ decision makers can be evaluated as:

$$\tilde{w}_j = \frac{1}{l} \left( \tilde{o}_{j1} \oplus \tilde{o}_{j2} \oplus \ldots \oplus \tilde{o}_{jl} \oplus \ldots \oplus \tilde{o}_{jl} \right),$$

(8)

where $\tilde{o}_j = (1/l) \sum_{t=1}^{l} \tilde{o}_{jt}$, $\tilde{p}_j = (1/l) \sum_{t=1}^{l} \tilde{p}_{jt}$, and $\tilde{q}_j = (1/l) \sum_{t=1}^{l} \tilde{q}_{jt}$.

3.3. Normalize performance of alternatives versus objective criteria

Suppose $\tilde{r}_j = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij})$ is the performance of alternative $i$ on criteria $j$. The normalized value $\tilde{s}_{ij}$ can then be denoted as:
\begin{align}
\tilde{x}_{ij} &= \left( \tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij} \right), j \in B \\
\tilde{x}_j &= \left( \frac{\tilde{a}_j \cdot \tilde{b}_j \cdot \tilde{c}_j}{\epsilon_j}, \frac{\tilde{a}_j \cdot \tilde{b}_j \cdot \tilde{c}_j}{\delta_j}, \frac{\tilde{a}_j \cdot \tilde{b}_j \cdot \tilde{c}_j}{\gamma_j} \right), j \in C
\end{align}

where \( \tilde{a}_j = \min_i \tilde{a}_i, \tilde{b}_j = \max_i \tilde{b}_i, i = 1, \ldots, m; j = 1, \ldots, n \)

3.4. Develop a membership function of each normalized weighted rating

The membership function of each final fuzzy evaluation value, i.e. \( \tilde{T}_j = \tilde{x}_j \otimes \tilde{w}_j, i = 1, \ldots, m; j = 1, \ldots, n \) can be developed by the interval arithmetic of fuzzy numbers. By Equations (2), (3), and (5), the \( \alpha \)-cuts of \( \tilde{T}_j = \tilde{x}_j \otimes \tilde{w}_j \), can be expressed as follows:

\begin{equation}
(\tilde{T}_j)^{\alpha} = (\tilde{x}_j \otimes \tilde{w}_j)^{\alpha} = [(\tilde{f}_j - \tilde{e}_j)(\tilde{p}_j - \tilde{o}_j)\alpha^2 + [\tilde{e}_j(\tilde{p}_j - \tilde{o}_j) + \tilde{o}_j(\tilde{f}_j - \tilde{e}_j)]\alpha + \\
\tilde{e}_j\tilde{o}_j,(\tilde{f}_j - \tilde{g}_j)(\tilde{p}_j - \tilde{q}_j)\alpha^2 + [\tilde{g}_j(\tilde{p}_j - \tilde{q}_j) + \tilde{q}_j(\tilde{f}_j - \tilde{g}_j)]\alpha + \tilde{g}_j\tilde{q}_j]
\end{equation}

We now have two equations to solve, namely:

\begin{align}
\tilde{A}_j\alpha^2 + \tilde{B}_j\alpha + \tilde{Q}_j - x = 0 \\
\tilde{C}_j\alpha^2 + \tilde{D}_j\alpha + \tilde{Z}_j - x = 0
\end{align}

where \( \tilde{A}_j = (\tilde{f}_j - \tilde{e}_j)(\tilde{p}_j - \tilde{o}_j) \)
\( \tilde{B}_j = [\tilde{e}_j(\tilde{p}_j - \tilde{o}_j) + \tilde{o}_j(\tilde{f}_j - \tilde{e}_j)] \)
\( \tilde{C}_j = (\tilde{f}_j - \tilde{g}_j)(\tilde{p}_j - \tilde{q}_j) \)
\( \tilde{D}_j = [\tilde{g}_j(\tilde{p}_j - \tilde{q}_j) + \tilde{q}_j(\tilde{f}_j - \tilde{g}_j)] \)
\( \tilde{Q}_j = \tilde{e}_j\tilde{o}_j \)
\( \tilde{Z}_j = \tilde{g}_j\tilde{q}_j \)

Only the roots in [0,1] will be retained in (11) and (12). The left and right membership functions \( f^{L}_{\tilde{T}_j} (x) \) and \( f^{R}_{\tilde{T}_j} (x) \) of \( \tilde{T}_j \) can be calculated as:

\begin{align}
f^{L}_{\tilde{T}_j} (x) &= \left\{ -\tilde{B}_j + [\tilde{B}_j^2 + 4\tilde{A}_j(x - \tilde{Q}_j)]^{1/2} \right\} / 2\tilde{A}_j \quad \text{with } \tilde{Q}_j \leq x \leq \tilde{Y}_j \\
f^{R}_{\tilde{T}_j} (x) &= \left\{ -\tilde{D}_j - [\tilde{D}_j^2 + 4\tilde{C}_j(x - \tilde{Z}_j)]^{1/2} \right\} / 2\tilde{C}_j \quad \text{with } \tilde{Y}_j \leq x \leq \tilde{Z}_j
\end{align}

For convenience, \( \tilde{T}_j \) is expressed as:

\( \tilde{T}_j = (\tilde{Q}_j, \tilde{Y}_j, \tilde{Z}_j; \tilde{A}_j, \tilde{B}_j; \tilde{C}_j, \tilde{D}_j), i = 1, \ldots, m; j = 1, \ldots, n \)

3.5. Defuzzification

In this paper, we employ the ranking approach put forth by Vincent and Dat (2014) to convert all the final evaluation values of each alternative from fuzzy to crisp values. The ranking process is outlined as follows:

Suppose there are \( n \) fuzzy numbers \( \tilde{A}_i, i = 1, 2, \ldots, n \), each with the left membership function \( f^{L}_{\tilde{A}_i} \) and right membership function \( f^{R}_{\tilde{A}_i} \). The left and right integral values of \( \tilde{A}_i \) are defined as:

\begin{align}
\tilde{S}^{L}_{\tilde{A}_i} &= \omega_i (\tilde{B}_i - \tilde{x}_{min}) - \int_{\tilde{x}_{min}}^{\tilde{T}_j} f^{L}_{\tilde{A}_i} (x) dx, \\
\tilde{S}^{R}_{\tilde{A}_i} &= \omega_i (\tilde{C}_i - \tilde{x}_{max}) + \int_{\tilde{x}_{max}}^{\tilde{T}_j} f^{R}_{\tilde{A}_i} (x) dx
\end{align}

where \( \tilde{x}_{min} = \inf \tilde{P}, \tilde{P} = U^{\tilde{A}_i}_{\tilde{A}_j}, \tilde{P}_j = \{ x \mid f^{L}_{\tilde{A}_i} (x) > 0 \}, \tilde{w}_j = \sup_x f^{L}_{\tilde{A}_i} (x). \) Both \( \tilde{S}^{L}_{\tilde{A}_i} \) and \( \tilde{S}^{R}_{\tilde{A}_i} \geq 0. \)
Then, the total integral value with index of optimism \( a \in [0,1] \) is defined as:

\[
\mathcal{S}_T^a (\tilde{A}_i) = \alpha \mathcal{S}_L^a (\tilde{A}_i) + (1 - \alpha) \mathcal{S}_R^a (\tilde{A}_i) \tag{17}
\]

### 3.6. Obtain the ranking values

This paper applies Vincent and Dat’s (2014) ranking method to defuzzify all the final fuzzy evaluation values \( \mathcal{T}_{ij} \).

From Equations (15) and (16), the left and right integral values of the fuzzy evaluation value, \( \mathcal{T}_{ij} \), are given by:

\[
\mathcal{S}_L (\mathcal{T}_{ij}) = Y_{ij} - \tilde{Q}_{ij_{\min}} - \int_{\tilde{A}_i} \left\{ -B_{ij} + [B_{ij}^2 + 4\tilde{A}_y(x - \tilde{Q}_{ij})]^{1/2} \right\} / 2 \tilde{A}_y \, dx \tag{18}
\]

\[
\mathcal{S}_R (\mathcal{T}_{ij}) = Z_{ij} - \tilde{Q}_{ij_{\min}} + \int_{\tilde{A}_i} \left\{ -D_{ij} - [D_{ij}^2 + 4\tilde{C}_y(x - \tilde{Z}_{ij})]^{1/2} \right\} / 2 \tilde{C}_y \, dx \tag{19}
\]

Then, the total integral value of \( \mathcal{T}_{ij} \), with index of optimism \( a \in [0,1] \) is calculated as:

\[
\mathcal{S}_T^a (\mathcal{T}_{ij}) = \alpha \mathcal{S}_R (\mathcal{T}_{ij}) + (1 - \alpha) \mathcal{S}_L (\mathcal{T}_{ij}) \tag{20}
\]

### 3.7. Calculation of \( \tilde{A}^+, \tilde{A}^- \), \( \tilde{d}_i^+ \) and \( \tilde{d}_i^- \)

The fuzzy positive-ideal solution (FPIS, \( \tilde{A}^+ \)) and fuzzy negative ideal solution (FNIS, \( \tilde{A}^- \)) are obtained as:

\[
\tilde{A}^+ = \max_i \{ \mathcal{S}_T^a (\mathcal{T}_{ij}) \} \tag{21}
\]

\[
\tilde{A}^- = \min_i \{ \mathcal{S}_T^a (\mathcal{T}_{ij}) \} \tag{22}
\]

The distance of each alternative \( \tilde{A}_i, i = 1, \ldots, m \) from \( \tilde{A}^+ \) and \( \tilde{A}^- \) is calculated as:

\[
\tilde{d}_i^+ = \sqrt{\sum_{j=1}^{n} (\mathcal{S}_T^a (\mathcal{T}_{ij}) - \tilde{A}^+)^2} \tag{23}
\]

\[
\tilde{d}_i^- = \sqrt{\sum_{j=1}^{n} (\mathcal{S}_T^a (\mathcal{T}_{ij}) - \tilde{A}^-)^2} \tag{24}
\]

where \( \tilde{d}_i^+ \) represents the shortest distance of alternative \( \tilde{A}_i \), and \( \tilde{d}_i^- \) represents the farthest distance of alternative \( \tilde{A}_i \).

### 3.8. Obtain the closeness coefficient

The closeness coefficient for each alternative, typically employed to establish the ranking order among all alternatives, is determined as follows:

\[
CC_i = \frac{\tilde{d}_i^-}{\tilde{d}_i^+ + \tilde{d}_i^-} \tag{25}
\]

A higher closeness coefficient signifies that an alternative is simultaneously closer to the PIS and farther from the NIS. This coefficient is instrumental in establishing the ranking order among all alternatives and pinpointing the best choice from a set of viable options.

### 4. Applying the proposed method to solve a banking performance evaluation problem

In this section, we put the proposed method into practice to address a banking performance evaluation challenge, showcasing the practicality of the approach. Following an initial screening process, we have selected five banks in Vietnam (\( A_1, A_2, A_3, A_4 \) and \( A_5 \)) for a more detailed assessment. A committee comprising three decision-makers (\( D_1, D_2, \) and \( D_3 \)) is
entrusted with evaluating these five banks based on five criteria: capital sufficiency \( (C_1) \), asset quality \( (C_2) \), earnings and profitability \( (C_3) \), liquidity \( (C_4) \), and interest rate sensitivity \( (C_5) \). The computational process is outlined as follows:

**Step 1. Aggregate ratings of alternatives versus criteria**

Let’s assume that the decision-makers employ a linguistic rating set as follows: VP (Very Poor) = (0.0, 0.1, 0.2), P (Poor) = (0.1, 0.3, 0.5), F (Fair) = (0.3, 0.5, 0.7), G (Good) = (0.6, 0.7, 1.0), and VG (Very Good) = (0.8, 0.9, 1.0) to assess the appropriateness of the banks under each criterion. In Table 1, you can find the combined suitability ratings for five banks concerning various criteria, as evaluated by three decision-makers using Equation (8).

Table 1 Ratings of alternatives versus criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Banks</th>
<th>Decision makers</th>
<th>( \bar{F}_{ij} )</th>
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<tbody>
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**Step 2. Aggregate the importance weights**

Let’s consider that the decision-makers utilize a linguistic weighting set as follows: UI (Unimportant) = (0.0, 0.1, 0.3), OI (Ordinary Important) = (0.2, 0.3, 0.4), I (Important) = (0.3, 0.5, 0.7), VI (Very Important) = (0.7, 0.8, 0.9), and AI (Absolutely Important) = (0.8, 0.9, 1.0) to assess the significance of all the criteria. The importance weights for the five criteria, as provided by three decision-makers, are presented in Table 2. The combined criteria weights, calculated using Equation (9), are presented in the last column of Table 2.

Table 2 The importance weights of the criteria and the aggregated criteria weights.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Decision makers</th>
<th>( \bar{F}_{ij} )</th>
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<tbody>
<tr>
<td>( C_1 )</td>
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<tr>
<td>( C_2 )</td>
<td>VI</td>
<td>VI</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>VI</td>
<td>I</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>I</td>
<td>OI</td>
</tr>
</tbody>
</table>

**Step 3. Develop the membership function of each normalized weighted rating**

The final fuzzy evaluation value of each bank can be obtained by Equations (11) - (15).

**Step 4. Defuzzification**
Using Equations (19) - (21), the left, right and total integral values of each Bank with \( \alpha = 1/2 \) can be obtained, as shown in Table 3.

### Table 3 The left, right and total integral values of each bank.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Banks</th>
<th>( \bar{S}_L(\bar{A}) )</th>
<th>( \bar{S}_R(\bar{A}) )</th>
<th>( \bar{S}_{l/2}(\bar{A}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>A1</td>
<td>0.52</td>
<td>0.75</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>0.46</td>
<td>0.71</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>0.39</td>
<td>0.64</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>0.27</td>
<td>0.50</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>A5</td>
<td>0.26</td>
<td>0.48</td>
<td>0.37</td>
</tr>
<tr>
<td>C2</td>
<td>A1</td>
<td>0.52</td>
<td>0.78</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>0.24</td>
<td>0.47</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>0.24</td>
<td>0.47</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>0.23</td>
<td>0.46</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>A5</td>
<td>0.34</td>
<td>0.58</td>
<td>0.46</td>
</tr>
<tr>
<td>C3</td>
<td>A1</td>
<td>0.39</td>
<td>0.61</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>0.29</td>
<td>0.52</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>0.47</td>
<td>0.66</td>
<td>0.56</td>
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<tr>
<td></td>
<td>A4</td>
<td>0.14</td>
<td>0.34</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>A5</td>
<td>0.19</td>
<td>0.45</td>
<td>0.32</td>
</tr>
<tr>
<td>C4</td>
<td>A1</td>
<td>0.22</td>
<td>0.46</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>0.22</td>
<td>0.46</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>0.27</td>
<td>0.50</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>0.07</td>
<td>0.25</td>
<td>0.16</td>
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<td></td>
<td>A5</td>
<td>0.23</td>
<td>0.42</td>
<td>0.32</td>
</tr>
<tr>
<td>C5</td>
<td>A1</td>
<td>0.18</td>
<td>0.39</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>0.13</td>
<td>0.33</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>0.07</td>
<td>0.24</td>
<td>0.16</td>
</tr>
</tbody>
</table>

**Step 5.** Calculate \( \bar{A}^+, \bar{A}^-, \bar{d}^+_j \) and \( \bar{d}^-_j \)

As shown in Table 4, the distance of each bank from \( \bar{A}^+ \) and \( \bar{A}^- \) can be calculated by Equations (22) - (25).

### Table 4 The distance measurement.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>( \bar{d}^+_j )</th>
<th>( \bar{d}^-_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.789</td>
<td>0.778</td>
</tr>
<tr>
<td>A2</td>
<td>0.909</td>
<td>0.619</td>
</tr>
<tr>
<td>A3</td>
<td>0.861</td>
<td>0.673</td>
</tr>
<tr>
<td>A4</td>
<td>0.959</td>
<td>0.56</td>
</tr>
<tr>
<td>A5</td>
<td>1.24</td>
<td>0.29</td>
</tr>
</tbody>
</table>

**Step 6.** Obtain the closeness coefficient

Closeness coefficients for the alternatives can be computed using Equation (26), as demonstrated in Table 5. Consequently, the ranking order of the five banks is \( A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_5 \). So, the best bank is \( A_1 \).

### Table 5 Closeness coefficients of alternatives.

<table>
<thead>
<tr>
<th>Banks</th>
<th>Closeness coefficient</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.497</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>0.405</td>
<td>3</td>
</tr>
<tr>
<td>A3</td>
<td>0.439</td>
<td>2</td>
</tr>
<tr>
<td>A4</td>
<td>0.367</td>
<td>4</td>
</tr>
<tr>
<td>A5</td>
<td>0.192</td>
<td>5</td>
</tr>
</tbody>
</table>

The research findings indicate that the proposed model is effective in assessing the performance of banks. Compared to existing decision-making models, the study developed membership functions for the final fuzzy evaluation values of alternatives using interval arithmetic for fuzzy numbers. A new method for ranking fuzzy numbers based on integral values was utilized to simplify the aggregation of fuzzy numbers for defuzzification of the weighted ratings. A closeness coefficient
was employed to determine the ranking order of alternatives by calculating their distances to both the positive and negative ideal solutions.

5. Conclusions

In this paper, a novel fuzzy TOPSIS approach was presented for addressing the challenge of evaluating banking performance. The proposed approach involved consolidating and normalizing the ratings and weights assigned by decision makers to facilitate comparison on a standardized scale. To simplify the calculation of distances between each alternative and the positive/negative ideal solutions, the normalized weighted ratings were defuzzified using the proposed integral value based ranking method instead of relying on complicated calculations involving fuzzy numbers. By defining a closeness coefficient, the proposed approach is able to establish the ranking order of alternatives. To illustrate the effectiveness of the proposed approach, a numerical example was provided. Furthermore, it should be noted that the proposed approach is not limited to banking performance evaluation and can be extended to address various MCDM problems such as sustainable bank performance evaluation, green supplier selection, etc. Additionally, further research could apply different sets of criteria to evaluate banking performance.

Ethical considerations

Not applicable.

Conflict of Interest

The authors declare no conflicts of interest.

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References


